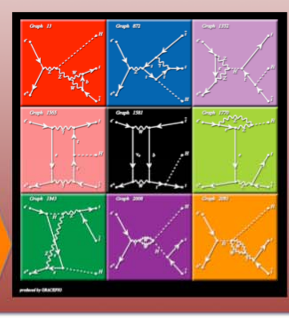
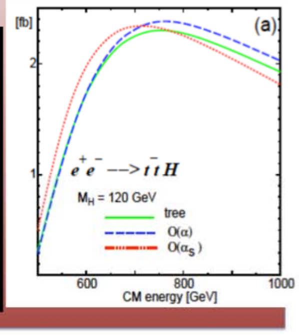
Acceleration of Feynman loop integrals in high-energy physics on many core GPUs

F Yuasa[1], T Ishikawa[1], N Hamaguchi[1], T Koike[2], N Nakasato[3]
1, High Energy Accelerator Research Organization (KEK), 1-1 Oho, Tsukuba, Ibaraki 305-0801, Japan
2, Seikei University, Musashino, Tokyo 180-8633, Japan
3, University of Aizu, Aizu-Wakamatsu, Fukushima 965-8580, Japan

Much of the success of the Standard Model, rests on the result of the various precision measurements.
These precision measurements required the knowledge of higher order quantum corrections.
In the diagrammatic Feynman approach followed by most of the automated systems,
the cross sections can be obtained by computing directly the squared of matrix elements or amplitudes.

Calculation by Hand → More Accuracy → More Complexity of the reaction → Numerical Method by Computer





Higher order corrections

Feynman loop integrals for L-loops with N internal line

$$(-1)^N\left(\frac{1}{4\pi}\right)^{nL/2}\Gamma(N-nL/2)\int_0^1\prod_{i=1}^{N}dx_i\delta(1-x_1\cdots-x_N)\frac{C^{N-n(L+1)/2}}{(D-i\varepsilon C)^{N-nL/2}}$$

D and C functions are polynomials of x_i

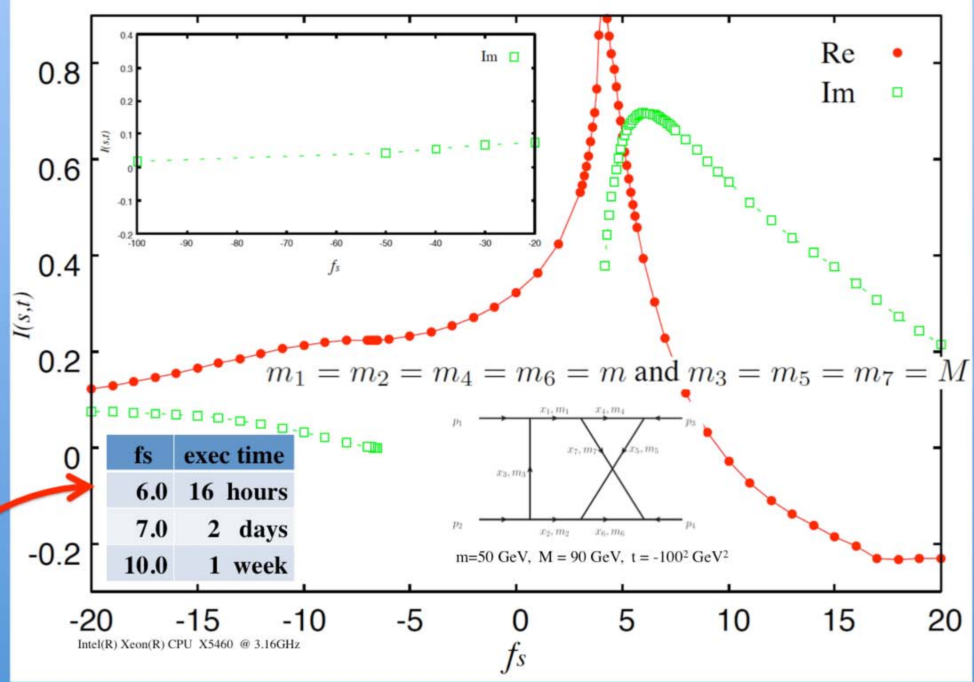
Direct Computation Method (DCM)

Combination of iterative numerical multivariate Integration and extrapolation

DQ-DCM: QUADPACK
DE-DCM: Double Exponential formulas
+ Wynn's epsilon algorithm

Fully numerical method
Pros: flexibility
various masses, multi legs, multi loops
good match with automated systems
Cons: long execution time

$$I=-\int_0^1 dx_1\,dx_2\,dx_3\,dx_4\,dx_5\,dx_6\,dx_7\,\delta(1-\sum_{\ell=1}^{7}x_\ell)\,\frac{C}{(D-i\epsilon C)^3}$$



fs	exec time
6.0	16 hours
7.0	2 days
10.0	1 week

Numerical results of Two-loop non-planar box with masses
Ref. F.Yuasa et al. Computer Physics Communications 183 (2012) 2136.

Acceleration of DCM on GPU's

	HD5870	HD6970
Clock [MHz]	850	880
Stream processor	1600	1536
SP FP [GFLOP]	2740	2732
DP FP[GFLOP]	544	683

Software env.: LSUMP, Goose

Ref. N.Nakasato et al. IEEE International Conference on Cluter Computing and Workshops, 2009, pp1–9.

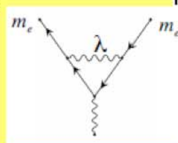
1st example: One-loop vertex (3 legs)

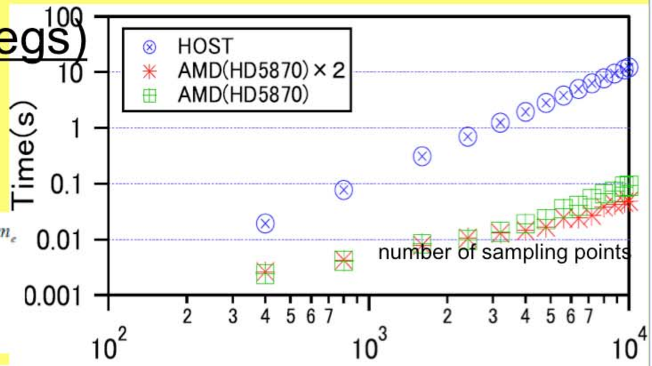
$$I=\int_0^1 dx\int_0^{1-x}dy\left[\frac{D}{D^2+\varepsilon^2}+i\,\frac{\varepsilon}{D^2+\varepsilon^2}\right]$$

$$D=-xys+(x+y)^2m_e^2+(1-x-y)\lambda^2$$

$s=500^2\,\mathrm{GeV}^2\quad \lambda=90\,\mathrm{GeV}$

$m_e=0.5\times10^{-3}\,\mathrm{GeV}$

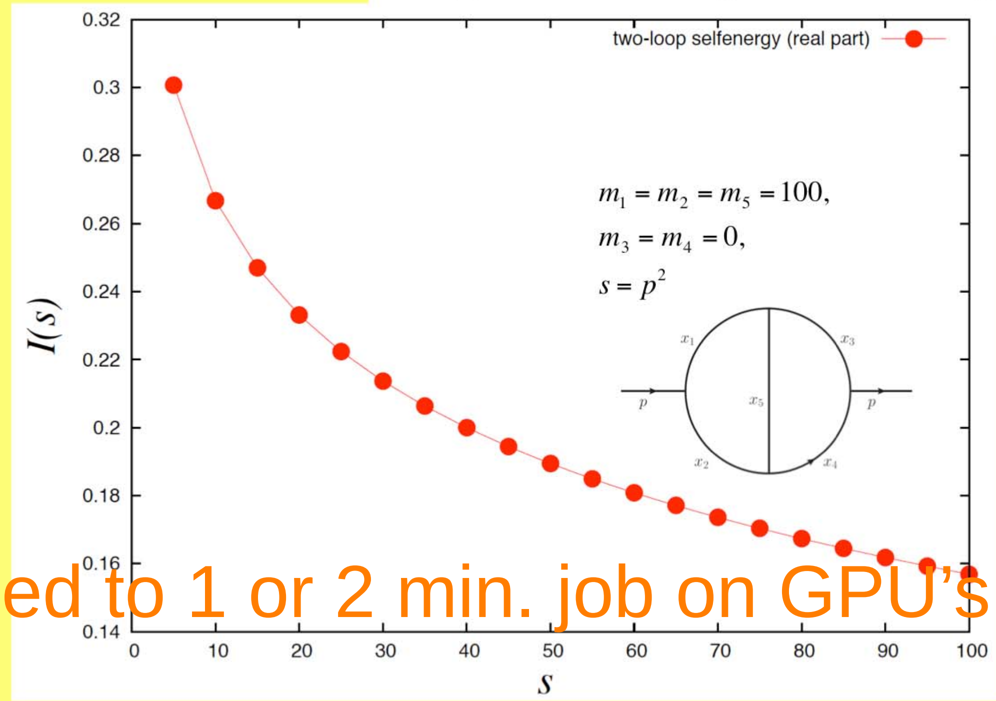




2nd example: Two-loop selfenergy (2 legs)

$$I=\int_0^1 dx_1dx_2dx_3dx_4dx_5\delta(1-\sum_{i=1}^{5}x_i)\frac{1}{CD}$$

$$D=-s(x_5(x_1+x_3)(x_2+x_4)+(x_1+x_2)x_3x_4+(x_3+x_4)x_1x_2)$$
$$+C\tilde{M}^2,$$
$$C=(x_1+x_2+x_3+x_4)x_5+(x_1+x_2)(x_3+x_4),$$
$$\tilde{M}^2=\sum_{i=1}^{5}x_im_i^2.$$



1 day job on host is reduced to 1 or 2 min. job on GPU's

Future plan:
Extension to more complex loop integrals such as
- Two-loop vertex (3 legs),
- Two-loop box (4 legs),
- Three-loop selfenergy (2 legs).

HOST computer	Two-loop selfenergy (s=5) [sec]
AMD Phenon II X6 1090T (3.2GHz)	79903.860
AMD FX-8150 (3.6GHz)	96984.307

s	size	HD5870 x 2 [sec]	HD6970 x 4 [sec]
-1	2048	1600.489	882.288
5	1024	122.551	55.667